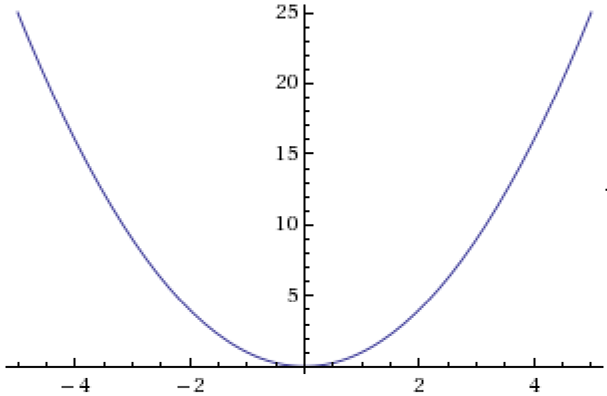
II. Derivatives

A. Concept of the Derivative (Timmy Nykamp)

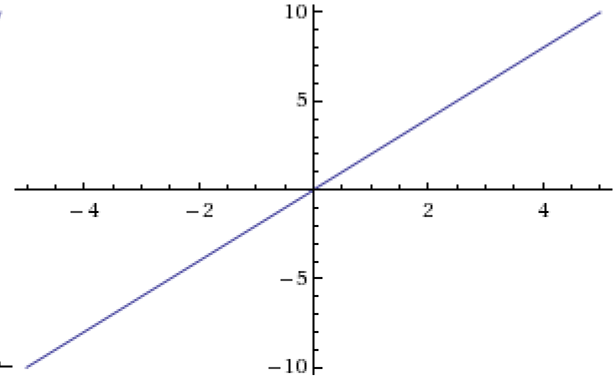
1. (2.1, 2.2) Derivative presented graphically, numerically, and analytically:

Graphically:

$f(x)=x^2$



$f'(x)=2x$



Numerically: $f(x)=2x^2 + 2x + 2$ $f'(x)=4x + 2$

	-1	**1**
f(x)	2	6
f'(x)	-2	6

Analytically:

Constant Rule: $f(x)=a$ $f'(x)=0$
Power Rule: $f(x)=x^n$ $f'(x)=nx^{(n-1)}$
Sum Rule: $f(x)=g(x)+h(x)$ $f'(x)=g'(x)+h'(x)$
Difference Rule: $f(x)=g(x) - h(x)$ $f'(x)=g'(x) - h'(x)$
Basic Trig Functions: $f(x)=\sin x$ $f'(x)=\cos x$
$f(x)=\cos x$ $f'(x)=-\sin x$
Constant Multiple Rule: $f(x)=a\, g(x)$ $f'(x)=a\, g'(x)$

2. (2.1) Derivative interpreted as an instantaneous rate of change:

<u>Instantaneous Rate of Change: The instantaneous rate of change on a point of a graph is the slope of the graph at that point. This is found by finding the slope of the tangent line at that point.</u>

The derivative can be interpreted as the instantaneous rate of change because the

derivative represents the slope of the tangent line at a certain point of the function. The slope is the rise over run, which is the rate of change, dy/dx; change in y (rise) over change in x (run).

3. **(2.1) Derivative defined as the limit of the difference quotient:**

The limit definition of the derivative is $f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

The limit definition of the derivative is that the derivative equals the limit as Δx approaches 0 of the function.

4. **(2.1) Relationship between differentiability and continuity:**

Differentiability: A function is differentiable if it doesn't have any holes, vertical tangent lines, or corners in the interval. Differentiability implies continuity.

Continuous Function: A function that does not have any holes, vertical asymptotes or gaps. Summarily, a function that is continuous is a graph that is defined for every point in the interval. Continuity does not necessarily imply differentiability.

As you can see by these definitions of continuity and differentiability, continuity means that the graph of a function is connected at all parts of the interval, but differentiability is a little bit different because it means that as well as being continuous, differentiability implies that there are no corners in the interval.

B. Derivative at a Point (Timmy Nykamp)

1. **(2.1) Slope of a curve at a point:**

The derivative at a point represents the instantaneous rate of change, or slope, at that point. You can find the derivative at a point as long as there is not a vertical asymptote, corner, or gap (usually caused by a vertical asymptote) in the graph of the function.

Ex 1| Find the instantaneous rate of change of the function $f(x)=3x^2$ at (1,3).

The derivative at the point (3,3) can be found by finding the derivative of the function and then putting the value for the function into the derivative of the function. The value of this is the instantaneous rate of change of the function.

$$f(x)=3x2 \to f'(x)=6x \to f'(1)=6(1) \to f'(1)=6$$

Therefore the instantaneous rate of change of the function $f(x)=3x^2$ at (1,3) is 6. This is also the slope of the function at that point.

Ex 2| Find the instantaneous rate of change of the function y=x/(x-1) when x=1.
This function has a vertical asymptote at x=1. This means that at this value of x, we can't find the derivative because the slope of a vertical line is undefined.

Ex 3| Find the slope of the function y=|x| at (0,0).
This function has a corner at the point (0,0). At a corner, it is impossible to find the derivative, so at the point (0,0), we cannot find the derivative, or instantaneous rate of change.

Ex 4| Find the instantaneous rate of change of the function f(x)={x^2 , x $\leq$ 0}
{x , x > 0}
This function is a piecewise function that has a function for the values of 0 and less and a function for the values greater than 0. This function has a corner at the point (0,0) and therefore is not differentiable at that point. Thus, we cannot find the instantaneous rate of change at that point.

2. (2.1) Tangent line to a curve at a point:

Tangent Line: A tangent line on a curve of a graph is a line that passes through the graph at a certain point. The slope of the tangent line equals the slope of the graph at that point.

The tangent line to a curve at a point represents the slope of the curve at that point. To find the equation of the tangent line at a point on the graph of a function, first you find the derivative of the function and then plug the ordered pair into the derivative of the function to find the derivative at the point. Then plug this value, the slope of the function at the given point, into the point-slope equation to find the equation of the tangent line to a graph at a given point.

Ex 1| Find the equation of the line tangent to the graph of f(x)=x^2 at the point (1,1)
First we find the derivative of the function. f(x)=x^2 → f'(x)=2x. Next we plug in the ordered pair into the equation. f'(1)=2(1) → f'(1)=2. We find that the slope at (1,1) is 2. Then we plug the ordered pair and the slope of the tangent line into the point-slope form of a line.
y-(1)=2(x-(1)) → y-1=2x-2 → y=2x-1. Thus, we find that the equation of the tangent line to the function f(x)=x^2 at the point (1,1) is y=2x-1.

3. (3.9) Local linear approximation:

Local Linear Approximation: A local linear approximation is a way to approximate the change in y of a function when the change in x is very small.

Applications of this are ways to approximate absolute error and percent error.

Local linear approximations can tell you approximately what the slope of the tangent line is at a point. This is accomplished by using the equation f'(x)=dy and then transforming it by multiplying by dx. The equation then becomes dy=f'(x)dx, which is the equation we use to approximate the slope of a tangent line at a point.

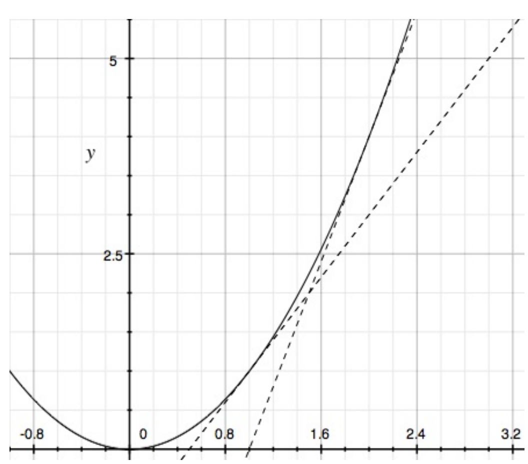
Ex 1| Find the slope of the tangent line at (1,1) of the equation f(x)=x^2.
The slope of the tangent line at this point can be found using the method of local linear approximations. First we differentiate the equation, finding that f'(x)=2x. Since f'(x)=dy/dx, we can multiply by dx to get dy=2xdx. Then we find the slope of the tangent line by reasoning that since dx goes to zero as we make the interval smaller and smaller, then dy becomes equal to 2.

4. (2.1) Instantaneous rate of change as the limit of average rate of change:
Instantaneous rate of change is the slope of the tangent line at a point of the graph of a function. This can be found by finding the limit of the average rate of change of a function as it approaches the value of the instantaneous rate of change. This is indicated by the equation

$$\lim_{x \to c} f(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

where c is the value of the desired instantaneous rate of change.

5. (2.1) Approximate rate of change from graphs and tables of values:
You can show the approximate rate of change from graphs and tables of values. Graphs show the approximate rate of change graphically, but tables of values show approximate rate of change numerically in an organized manner. Since the rate of change can be shown by lines tangent to the graph at certain points, graphs can show the approximate rate of change very well with multiple tangent lines.
Graph: f(x)=x^2



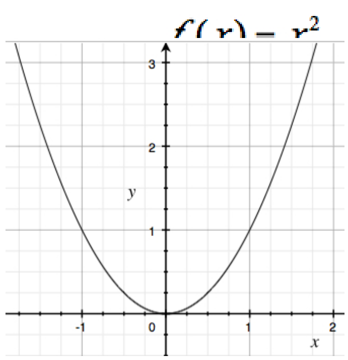
f(x)=2x^2 + 2x + 2

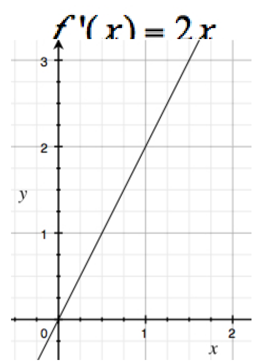
f’(x)=4x + 2

	-3	**-2**	**-1**	**0**	**1**	**2**	**3**
f(x)	14	6	2	2	6	14	26
f’(x)	-10	-6	-2	2	6	10	14

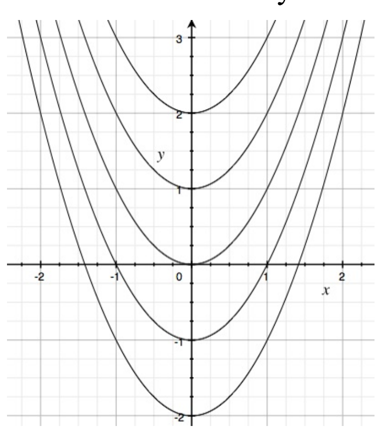
C. Derivative as a Function (Chris Poenaru)

1. **(3.1, 3.2, 3.3, 3.6) Corresponding characteristics of graphs of *f* and *f’*.**
    - *f(x)* is the function, where the y values are obtained by a function defined with respect to x. *f’(x)* is the derivative of *f(x)*, where the y values correspond to the tangent slopes at those points.
    - One can use *f(x)* to find the graph of *f’(x)*, and similarly use the graph of *f’(x)* to find the graph of *f(x)*. However, as the derivative eliminates terms with no x-variable, the position on the y-axis cannot be determined unless extra information is provided.





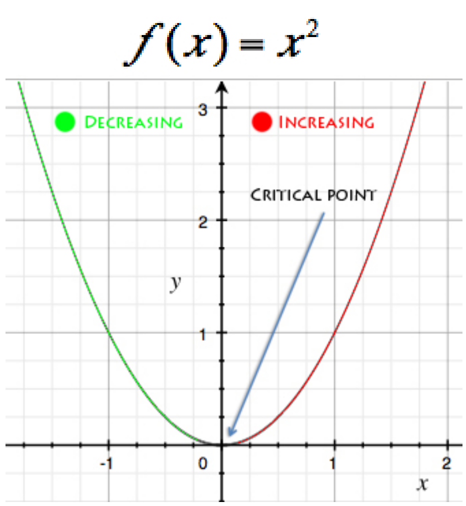
From the graphs above, one can see that you can accurately find the graph of the derivative of a function. However, going back to the function will give you and indeterminable y values, as +C is added on.

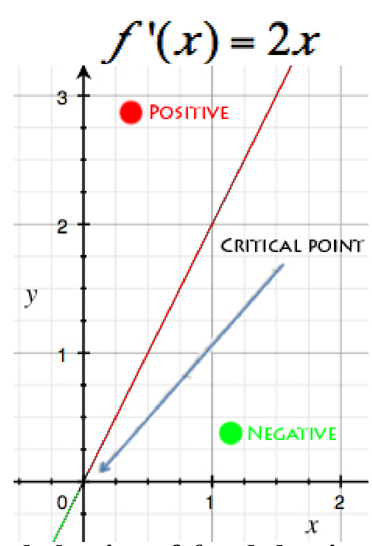


$f'(x) = 2x + c$

This graph is formed when *f(x)* is graphed from the derivative of *f'(x)*.

- Looking at the graph of either *f(x)* or/and *f'(x)*, one can gather information about the the other graph (*f(x)* or *f'(x)*).
- Looking at the graph of *f(x)*:
  i. The region of the graph where *f(x)* is increasing is where *f'(x)* is positive.
  ii. The region of the graph where *f(x)* is decreasing is where *f'(x)* is negative.
  iii. A shift in *f(x)* from increasing to decreasing (or vice versa) marks a critical point, and denotes a zero in the graph of *f'(x)*. However, if this is a sharp point (as in the graph *f(x)*= abs(*x*)), this will result in a non-differentiable point, and a hole or gap in the graph of *f'(x)*.
- Looking at the graph of *f'(x)*:
  i. The region of the graph where *f'(x)* is increasing is where *f(x)* is positive.
  ii. The region of the graph where *f'(x)* is decreasing is where *f'(x)* is negative.
  iii. The point(s) when *f'(x)* = 0 are extrema on the graph of *f(x),* either a maximum or minimum. It also marks a change from increasing to decreasing (or vice versa) of the graph of *f(x)*.



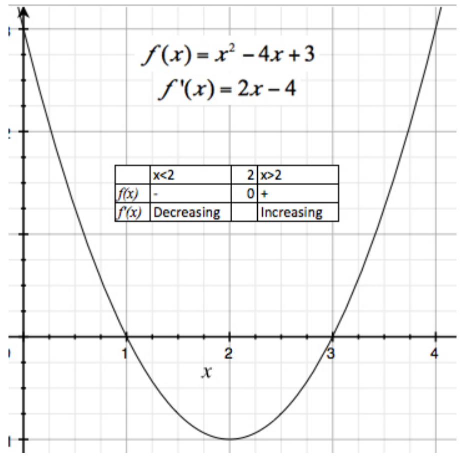


(3.3)

2. **Relationship between the increasing and decreasing behavior of *f* and the sign of *f'*.**

- In an increasing function, the slope is positive. In a decreasing function, the slope is negative.
- Let *f* be continuous on [a,b] and differentiable on (a,b).
  i. If $f'(x) > 0$, $\forall x \in (a,b)$, then *f* is increasing on [a,b].
  i. If $f'(x) < 0$, $\forall x \in (a,b)$, then *f* is decreasing on [a,b].
  ii. If $f'(x) = 0$, $\forall x \in (a,b)$, then *f* is constant on [a,b].

∀ = for all. ∈ = contained in

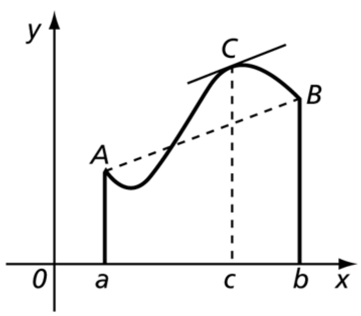


3. **(3.2) The Mean Value Theorem and its geometric interpretation.**
   - If *f* is continuous on [a,b] and differentiable on (a,b), then ∃ c ∈ (a,b) ∋

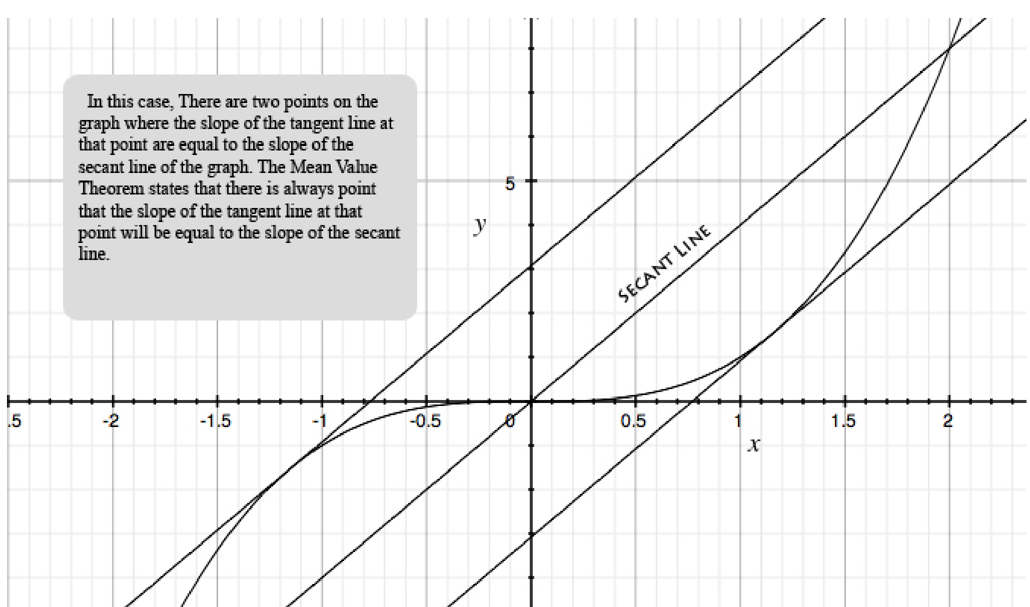
     $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

     ∃ = there exists. ∈= contained in. ∋= such that.

   - There is some point c where the slope of the tangent line is equal to the slope of the secant line through the endpoints.



The slope of the tangent line C is equal to the slope of the secant line AB.



4. **(6.2) Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.**
    - Based on verbal statements, one can write and solve differential equations.
    - Equations can be proportional, inversely proportional, or jointly proportional to a statement.
        i. Proportional is a direct relation. y=kx
        ii. Inversely proportional is a inverse relation. y=k/x
        iii. Joint variation is a direct relation with two or more quantities. y=kxz
    - Examples:
        i. The rate of change of *Q* with respect to *t* is inversely proportional to the square of *t.*
        dQ/dt = k/t^2
        ii. The rate of change of *N* with respect to *s* is proportional to 250-s.
        dN/ds = k(250-s)
        iii. The rate of change of *y* with respect to *x* is jointly proportional to *x* and *L-y*.
        dy/dx= x(L-y)*

<u>D. Second Derivatives (Harim Ahn)</u>

1. Corresponding Characteristics of the Graphs of *f, f', f''*. (3.3, 3.4)
    - <u>*f* and *f'*</u>

→*f* is simply a function where y values are obtained by a function defined with respect to x *(f(x))*

→*f'* is the derivative of *f* and the individual y-values correspond to the tangent slopes at those points

→When *f'* is positive, that means that f is increasing at that point and when *f'* is negative it means that f is decreasing at that point

→When *f'* is equal to zero it means that *f* has a zero at that point, either a maximum or a minimum

→When *f'* does not exist, it means that f is not differentiable at that point; for example the sharp point on an absolute value graph is non-differentiable and would not appear on the graph of *f'*

    - <u>*f'* and *f''*</u>

→*f"* is the derivative of *f'* and the individual y-values correspond to the tangent slopes at those points

→When *f"* is positive, that means that *f'* is increasing at that point and when *f"* is negative it means that *f'* is decreasing at that point

→When *f"* is equal to zero it means that *f'* has a zero at that point, either a maximum or a minimum

→When *f"* does not exist, it means that *f'* is not differentiable at that point; for example the sharp point on an absolute value graph is non-differentiable and would not appear on the graph of *f"*

- *f and f"*

→*f''* is the second derivate of *f* and *f''* defines concavity

→If *f* is differentiable on the open interval *I*, the graph of *f* is concave upward if *f'* is increasing on the interval and concave downward on *I* if *f'* is decreasing on the interval

→If *f''* is positive at a point then *f* is concave up and if *f''* is negative at a point then *f* is concave down

→If *f''* is zero at a point it means there is a point of inflection there, in other words concavity of *f* changes at this point

Example: You are given the function $f(x) = 2x^3 - 3x^2$ and asked to find the maximums, minimums and any points of inflection. First you differentiate and find that $f'(x) = 6x^2 - 6x$ and then set it equal to zero and find all zeros. You then determine that it has zeros as 0 and 1, that means those are critical points and they are possible maximums or minimums. You then pick values on either side of your critical points to see if the function is decreasing or increasing. For example, plugging in -1 will tell you the function is increasing, plugging in 1/2 will tell you the function is decreasing, and plugging in 2 will tell you the function is increasing over that interval. Since it is going from increasing to decreasing at 0, it means it is a maximum and because it is going from decreasing to increasing at 1, it mean it is a minimum. Next you differentiate again to find points of inflection. You find $f''(x) = 12x - 6$ and it has zero(s) at 1/2. To verify this point as a point of inflection, check values on both sides to see if the concavity is changing. Plugging in 0 tells you the function is concave down before 1/2 and plugging in 1 tells you the function is concave up after 1/2, this verifies 0 as a point of inflection.

2. and 3. Relationship Between the Concavity of *f* and the sign of *f"* **AND** Points of Inflection as Places where Concavity Changes. (3.4)
   - Concavity is the rate of change of the slope.
   - Concavity is found with the second derivative, *f"*.
   - Zeroes or undefined points for the second derivative are called Potential Inflection Points, PIPs.
   - Using the Second Derivative Test, one can confirm if PIPs are Inflection points, or the point in which the curve changes concavity.

- If, by using the Second Derivative Test
    1. $f''(x)>0$ on (a,b), then it is concave up on (a,b)
    2. $f''(x)<0$ on (a,b), then it is concave down on (a,b)
    3. $f''(x)=0$ on (a,b), then you don't know anything.

a. Positive Slope, Concave Up
b. Negative Slope, Concave Up
c. Positive Slope, Concave Down
d. Negative Slope, Concave Down

<u>E. Applications of Derivatives (Group 3)</u>

<u>F. Computation of Derivatives (Harim Ahn)</u>

1. Knowledge of Derivatives of Basic Functions, Including Power, Exponential, Logarithmic, Trigonometric and Inverse Trigonometric Functions. (2.2, 2.3, 4.1, 5.1, 5.4, 5.5, 5.6,)
    - Differential Formulas

$\frac{d}{dx}[c] = 0$

$\frac{d}{dx}[k\,x] = k$

$\frac{d}{dx}[k\,f(x)] = k\,f'(x)$

$\frac{d}{dx}[x^n] = nx^{n-1}$

$\frac{d}{dx}[\sin(x)] = \cos(x)$

$\frac{d}{dx}[\cos(x)] = -\sin(x)$

$\frac{d}{dx}[\tan(x)] = \sec^2(x)$

$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$

$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$

$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$

$\frac{d}{dx}[arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[arctan(x)] = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}[arcsec(x)] = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}[arccos(x)] = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[arccot(x)] = -\frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}[arccsc(x)] = -\frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}[ln(x)] = \frac{1}{x}$

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[a^x] = (ln(a) \cdot a^x)$

$\frac{d}{dx}[log_a x] = \frac{1}{(ln(a)\cdot x)}$

2. Derivative Rules for Sums, Products and Quotients of Functions. (2.2, 2.3)
    - Sum Rule: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
    - Product Rule: $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
    - Quotient Rule: $\frac{d}{dx}[f(x) \div g(x)] = \frac{[f[?][?][?](x)\cdot g(x) - f(x)\cdot g[?][?][?](x)]}{g(x)^2}$

Example:

- $y = 4x^6-3x^5-10x^2+5x+16$

  $y' = 24x^5-15x^4-20x+5$
- $y = (3x^2-5)(2x^4-1)$

  $y' = 6x(2x^4-1)+(3x^2-5)(8x)$

3. Chain Rule and Implicit Differentiation. (2.4, 2.5)
    - Chain Rule: When an equation is derived respectively with its corresponding rule, one must multiply again by the derivative of the equation within a function.

$\frac{d}{dx}[f(g(x))] = f'(g(x))\, g'(x)$

Alternatively: $\frac{d}{dx}[f(u)] = f'(u)\, u'$ **or** $\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}$

Examples:

i. $\frac{d}{dx}[(2x^3-4x+1)^{60}] = 60\,(2x^3-4x+1)^{59}\,(6x^2-4)$
ii. $\frac{d}{dx}[3\sin(x^3+1)] = 3\cos(x^3+1)\, 3x^2 = 9x^2\cos(x^3+1)$
iii. $\frac{d}{dx}[\tan^2(x)] = \frac{d}{dx}[(\tan(x))^2] = 2(\tan(x))\sec^2(x)$

- <u>Implicit Differentiation:</u> Consider the equation $y^3+7y = x^3$

  → How does one find the slope ($\frac{dy}{dx}$) along this equation?

  →Notice that one cannot simplify into making the equation equal to a single variable.

  →One must solve this equation implicitly

  →One must do the derivative of each turn with respect to the independent variable.

Example:

i. $y^3+7y = x^3$

$\frac{d}{dx}[y^3] + \frac{d}{dx}[7y] = \frac{d}{dx}[x^3]$

$[3y^2 \frac{dy}{dx}] + [7\frac{dy}{dx}] = [3x^2 \frac{dx}{dx}]$

$\frac{dy}{dx}(3y^2+7) = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{(3y^2+7)}$

<u>Practice Problems:</u>

A-1 1. Sketch the graph of f(x)=x^2. Then estimate the slope of the graph at the points (-1,1) and (1,1).

2. Sketch the graph of f(x)=1/2x^2. Then complete the table by graphically estimating the slopes of the graph at the indicated points.

x	-2	-1.5	-1	-.5	0	.5	1	1.5	2

f(x)									
f'(x)									

A-2 1. Find the instantaneous rate of change of the function f(x)=3x^2-2x at the points (0,1) and (1,2).

2. Given the function f(x)=x^2+5, find the slopes of the tangent lines to the graph at the points (0,5) and (3,14).

A-3 1. The limit represents f'(c) for a function f and a number c. Find f and c.

$$\lim_{\Delta x \to 0} \left[\frac{5-3(1+\Delta x)]-2}{\Delta x} \right]$$

A-4 1. Sketch the graph of f(x)=$|x|$. Is the function continuous at f(0)? Is is differentiable at f(0)?

B-1 For practice problems 1-4, find the slope of the graph of the function at the given point. Use the derivative feature of a graphing utility to confirm your results.

1. f(x)=3x^3-6 (2,18)
2. f(x)=(2x+1)^2 (0,1)
3. f(x)=4sin(x)-x (0,0)
4. g(t)=2+3cos(t) (,-1)
5. Sketch the graph of f(x)=$|x|$. Then find the slope of the graph of the function at the given points. If not possible, explain why not.
   a) (0,0)
   b) (1,1)
   c) (-2,2)

B-2 1. Sketch the graph of f(x)=(x^2+2x)(x+1). Then find an equation of the tangent line to the graph of f at the point (1,6).

B-3 1. Find the equation of the line T to the graph of f at the given point. Use this linear approximation to complete the table.

x	1.9	1.99	2	2.01	2.1
f(x)					
T(x)					

B-4 1. The limit represents f’(c) for a function f and a number c. Find f and c.

$$f'(c) = \lim_{x \to 6}\left[\frac{-x^{2+36}}{x-6}\right]$$

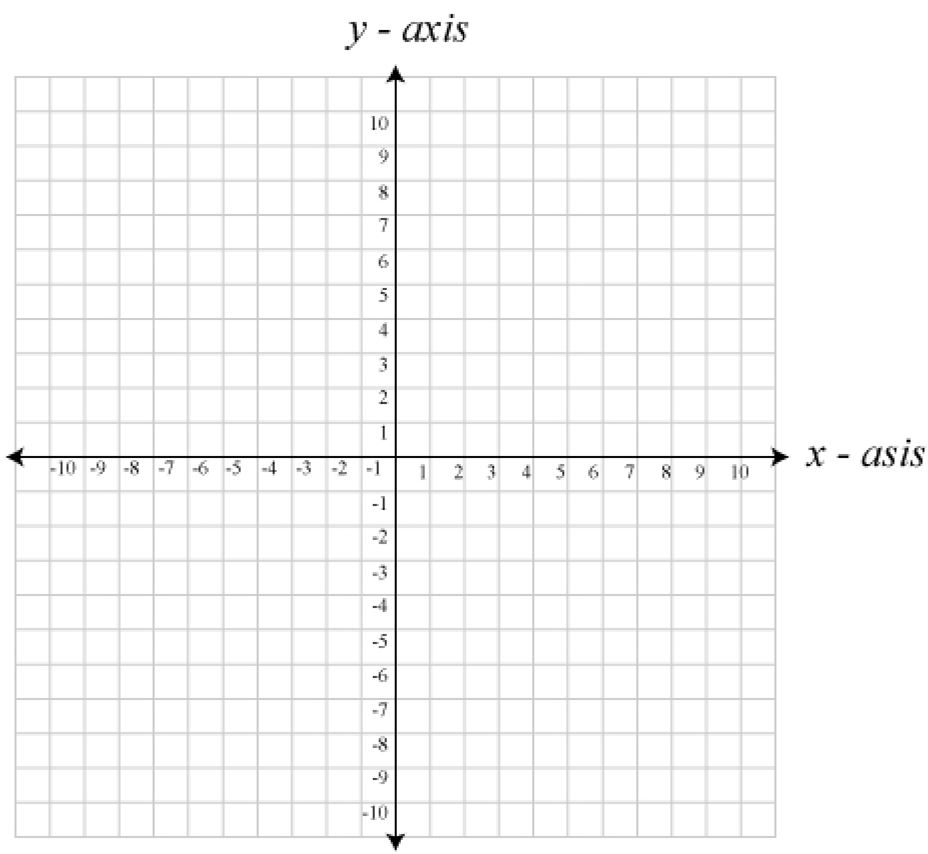
B-5 1. Sketch the graph of f(x)=(x-3)^2+4. Then find the approximate rate of change from the graph of f(x)=(x-3)^2+4 at the points (1,8) and (0,13).

C-1 Locate the absolute extrema of the function on the closed interval.

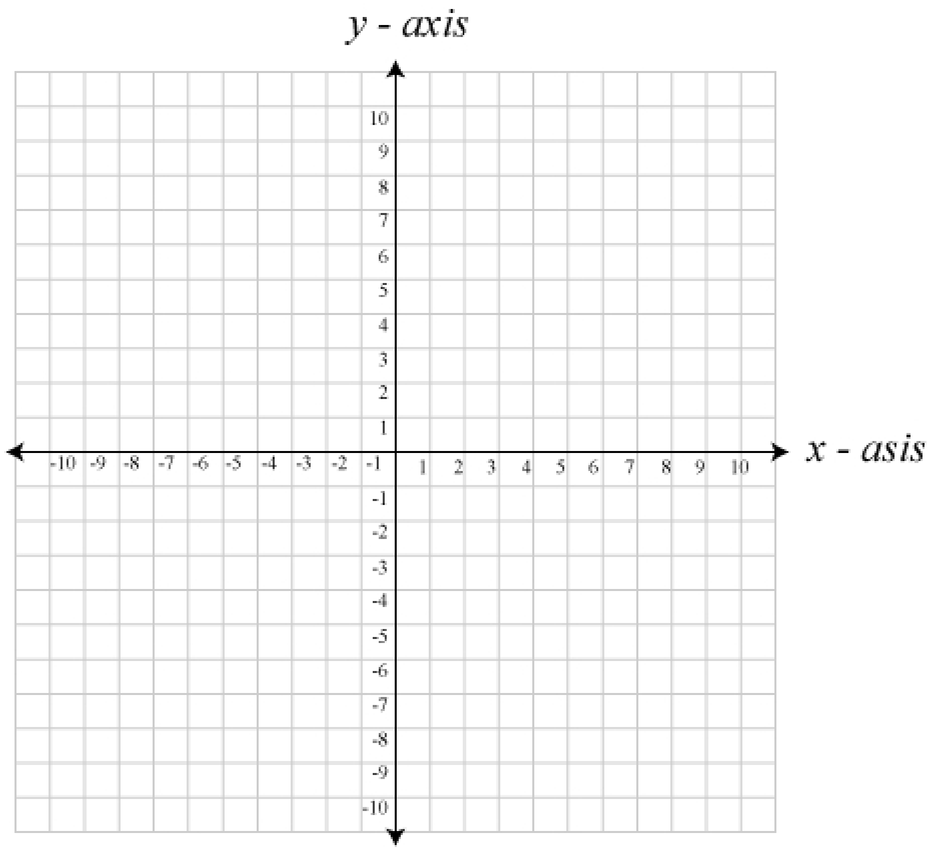
1. $f(x)=2(3-x), [-1,2]$
2. $f(x)= -x^2+3x, [0,3]$

Sketch the graph of the function. Then locate the absolute extrema of the function over the given interval.

1. $f(x)=\frac{3}{x-1}, (1,4]$



2. $f(x)=\frac{2}{2-x}$, [0,2)



C-2 Complete an analysis table of the graph of $f(x)=\frac{x^4+1}{x^2}$, locating the regions of increasing/decreasing, and critical points.

C-3 Consider the graph of the function $f(x)=x^2+1$.

a) Find the equation of the secant line joining the points (-1,2) and (2,5).

b) Use the Mean Value Theorem to determine a point c in the interval (-1,2) such that the tangent line at c is parallel to the secant line.

c) Find the equation of the tangent line through c.

d)Then use a graphing utility to graph f, the secant line, and the tangent line.

C-4 Write and solve the differential equation that models the verbal statement. Evaluate the solution at the specified value of the independent variable.

1. The rate of change of u is proportional to y. When x = 0, y = 4, and when x = 3, y = 10. What is the value of y when x = 6?
2. The rate of change of V is proportional to V. When t = 0, V = 20,000 and when

t = 4, V = 12,500. What is the value of V when t = 6?

Answer Key

A-1 1. Slope at (-1,1) = **-2**
Slope at (1,1) = **2**

2.

x	-2	-1.5	-1	-.5	0	.5	1	1.5	2
f(x)	.125	.222	.5	2	Error	2	.5	.222	.125
f’(x)	.125	.296	1	8	Error	-8	-1	-.296	-.125

A-2 1. f(x)=3x^2-2x → f’(x)=6x-2 → f’(0)=**-2**
f(x)=3x^2-2x → f’(x)=6x-2 → f’(1)=**4**
2. f(x)=x^2+5 → f’(x)=2x → f’(0)=**0**
f(x)=x^2+5 → f’(x)=2x → f’(3)=**6**

A-3 1. f is **f(x)=5-3x** from the top of $\lim_{\Delta x \to 0}\left[\frac{5-3(1+\Delta x)]-2}{\Delta x}\right]$.
c=**1** because this is the value that the limit goes to.

A-4 1. The graph of f(x)=$|x|$ is continuous at the point f(0). However, it is not differentiable at f(0) because it is a corner and therefore it has no slope at that point.

B-1 1. f(x)=3x^3-6 → f’(x)=9x^2 → f’(2)=**36**
2. f(x)=(2x+1)^2 → f’(x)=2(2x+1)(2) → f’(x)=4(2x+1) → f’(0)=**4**
3. f(x)=4sinx-x → f’(x)=4cosx-1 → f’(0)=**3**
4. f(x)=2+3cosx → f’(x)=-3sinx → f’( )=**0**
5. At (0,0) the graph has a corner. This means that it is not differentiable at this point. At (1,1) the graph has a slope of **1**. At (-2,2) the graph has a slope of **-1**.

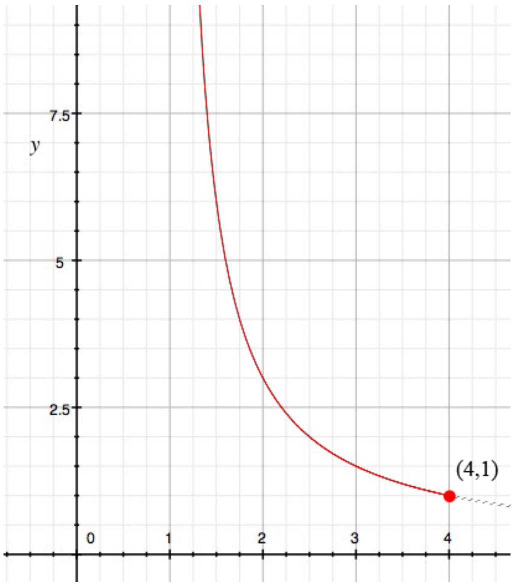
B-2 1. f(x)=(x^2+2x)(x+1) → f’(x)=3x^2+6x+2 → f’(1)=**11**.

B-3 1.

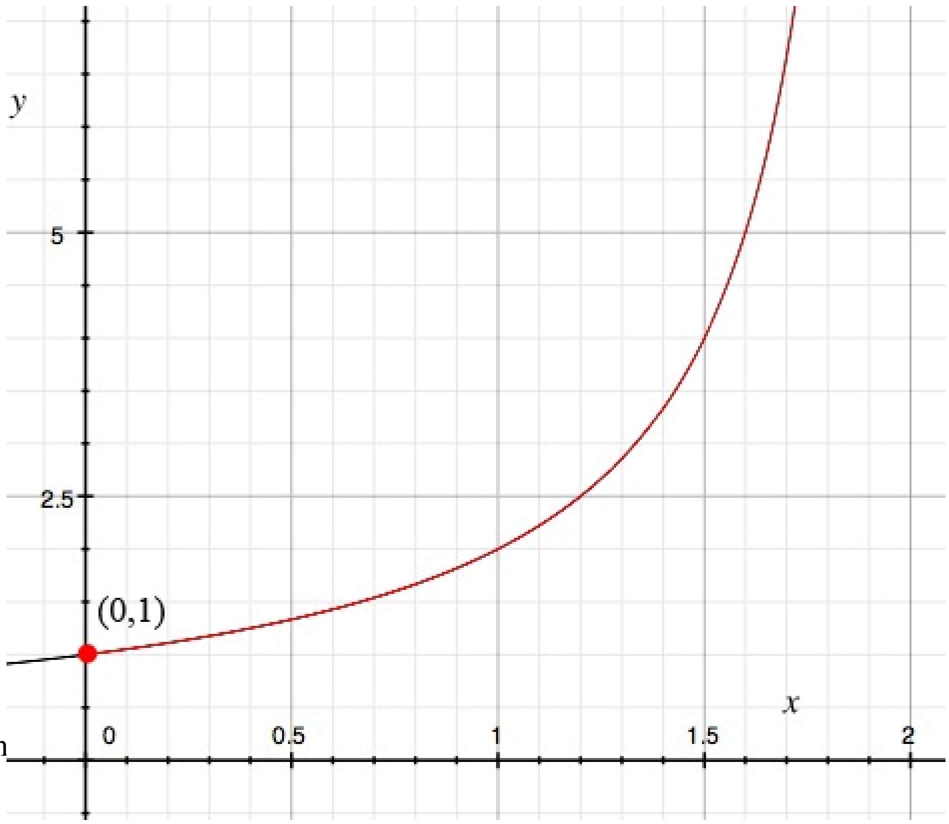
x	1.9	1.99	2	2.01	2.1
f(x)	1.662	1.515	1.5	1.485	1.361
T(x)	-1.75	-1.523	-1.5	-1.478	-1.296

B-4 1. Because of $\lim_{x \to c}\frac{f(x)-f(c)}{x-c}$, we can say that c is equal to **6** and f is equal to **(-x^2)**.

B-5 1. f(x)=(x-3)^2+4 → f’(x)=2(x-3) → f’(1)=-4
f(x)=(x-3)^2+4 → f’(x)=2(x-3) → f’(0)=-6

C-1 1. 

absolute extrema is the minimum at the endpo

2. 

The absolute extrem

C-2 1.
$$f(x)=\frac{x^4+1}{x^2}$$
$$f'(x)=\frac{2(x^4-1)}{x^3}$$

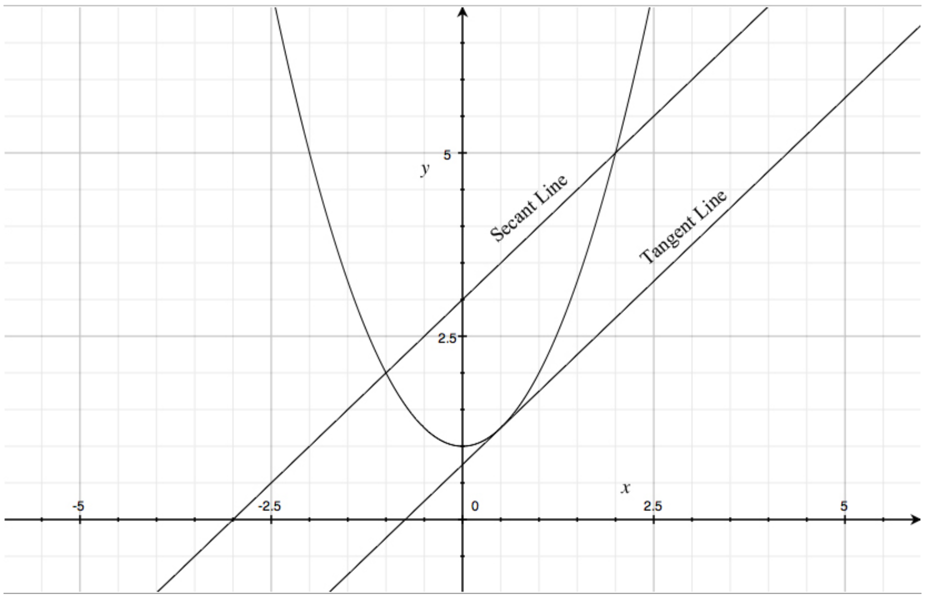
	-∞ < x < -1	x = -1	-1 < x < 0	x = 0	0 < x < 1	x = 1	1 < x < ∞
f(x)	+	2	+	undef	+	2	+
f'(x)	-	-4	+	undef	-	4	+

C-3 1. a) Secant line: x-y+3=0

b) c=1/2

c) Tangent line: 4x-4y+3=0

d)



C-4 1.

$$\frac{dy}{dx} = ky$$

$y = 4e^{0.3054x}$

y(6) = ~25

2. $\frac{dV}{dT} = kV$

$V = 20{,}000e^{-0.1175t}$

V(6) = ~9882